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17MATDIP31

Third Semester B.E. Degree Examination, July/August 2021
Additional Mathematics – I

Time: 3 hrs. Max. Marks: 100

Note: *Answer any FIVE full questions.*

1 a. Find the modulus and amplitude of $\frac{4+2i}{2-3i}$. **(06 Marks)**

b. Find a unit vector normal to both the vectors $4i-j+3k$ and $-2i+j-2k$. Find also sine of the angle between them. **(07 Marks)**

c. Show that $\left[\vec{a}+\vec{b}, \vec{b}+\vec{c}, \vec{c}+\vec{a}\right] = 2\left[\vec{a}, \vec{b}, \vec{c}\right]$. **(07 Marks)**

2 a. Express $(2+3i)+\frac{1}{1-i}$ in $x+iy$ form. **(06 Marks)**

b. Find the modulus and amplitude of $1+\cos\theta+i\sin\theta$. **(07 Marks)**

c. Find λ so that $\vec{a}=2\hat{i}-3\hat{j}+\hat{k}$, $\vec{b}=\hat{i}+2\hat{j}-3\hat{k}$ and $\vec{c}=\hat{j}+\lambda\hat{k}$ are coplanar. **(07 Marks)**

3 a. Find the n^{th} derivative of $e^{ax}\cos(bx+c)$. **(06 Marks)**

b. Find the angle of intersection of the curves $r=\sin\theta+\cos\theta$ and $r=2\sin\theta$. **(07 Marks)**

c. If, $z=f(x,y)$ where $x=e^{u}+e^{-v}$, $y=e^{-u}-e^{v}$. Prove that $x\frac{\partial z}{\partial x}-y\frac{\partial z}{\partial y}=\frac{\partial z}{\partial u}-\frac{\partial z}{\partial v}$. **(07 Marks)**

4 a. If $y=\tan^{-1}x$, then show that $(1+x^2)y_{n+2}+2(n+1)xy_{n+1}+n(n+1)y_n=0$. **(06 Marks)**

b. Find the pedal equation for the curve $\frac{2a}{r}=1+\cos\theta$. **(07 Marks)**

c. If, $u=x^2+y^2+z^2$, $v=xy+yz+zx$, $w=x+y+z$, then find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$. **(07 Marks)**

5 a. Obtain the reduction formula for $\int \cos^n x\,dx$. **(06 Marks)**

b. Using reduction formula, find the value of $\int_0^1 x^2(1-x^2)^{\frac{3}{2}}dx$. **(07 Marks)**

c. Evaluate $\int_{-1}^{1}\int_{0}^{2}\int_{x-z}^{x+z}(x+y+z)\,dx\,dy\,dz$. **(07 Marks)**

6 a. Evaluate $\int_0^{\pi} x \sin^8 x \, dx$. (06 Marks)

b. Evaluate $\int_0^1 \int_0^{\sqrt{1-y^2}} x^3 y \, dx \, dy$. (07 Marks)

c. Evaluate $\int_0^{\pi} x \sin^2 x \cos^4 x \, dx$. (07 Marks)

7 a. A particle moves along the curve $\vec{r} = 3t^2\hat{i} + (t^3 - 4t)\hat{j} + (3t + 4)\hat{k}$. Find the component of velocity and acceleration at $t = 2$ in the direction of $\hat{i} - 2\hat{j} + 2\hat{k}$. (06 Marks)

b. Find the angle between the tangents to the surface $x^2y^2 = z^4$ at (1, 1, 1) and (3, 3, −3). (07 Marks)

c. Find div $\vec{F}$ and curl $\vec{F}$ where $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$. (07 Marks)

8 a. Find the angle between the tangents and to the curve $\vec{r} = \left(t - \frac{t^3}{3}\right)\hat{i} + t^2\hat{j} + \left(t + \frac{t^3}{3}\right)\hat{k}$ at $t = \pm 3$. (06 Marks)

b. Find the directional derivative of $f = x^2yz + 4xz^2$ at (1,−2,−1) along $2\hat{i} - \hat{j} - 2\hat{k}$. (07 Marks)

c. Prove that $\text{div}(\text{curl}\,\vec{F}) = 0$. (07 Marks)

9 a. Solve $\frac{dy}{dx} = e^{3x-2y} + x^2e^{-2y}$. (06 Marks)

b. Solve $x^2y\,dx - (x^3 + y^3)dy = 0$. (07 Marks)

c. Solve $\frac{dy}{dx} - \frac{2y}{x} = x + x^2$. (07 Marks)

10 a. Solve $x\,dy - y\,dx = \sqrt{x^2 + y^2}\,dx$. (06 Marks)

b. Solve $(5x^4 + 3x^2y^2 - 2xy^3)dx + (2x^3y - 3x^2y^2 - 5y^4)dy = 0$. (07 Marks)

c. Solve $\frac{dy}{dx} - \frac{y}{x+1} = e^{3x}(x+1)$. (07 Marks)

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